1 Proof of General CIBP Termination

In the main paper, we discussed that the cascading Indian buffet process (CIBP) for fixed and finite $\alpha$ and $\beta$ eventually reaches a restaurant in which the customers choose no dishes. Every deeper restaurant also has no dishes. Here, we show a more general result, for IBP parameters that vary with depth: $\alpha^{(m)}$ and $\beta^{(m)}$.

Let there be an inhomogeneous Markov chain $\mathcal{M}$ with state space $\mathbb{N}$. Let $m$ index time and let the state at time $m$ be denoted $K^{(m)}$. The initial state $K^{(0)}$ is finite. The probability mass function describing the transition distribution for $\mathcal{M}$ at time $m$ is given by the following equation:

$$p(K^{(m+1)} = k \mid K^{(m)}, \alpha^{(m)}, \beta^{(m)}) = \frac{1}{k!} \exp \left\{-\alpha^{(m)} \sum_{k' = 1}^{K^{(m)}} \frac{\beta^{(m)}}{k' + \beta^{(m)}} - 1 \right\} \left(\alpha^{(m)} \sum_{k' = 1}^{K^{(m)}} \frac{\beta^{(m)}}{k' + \beta^{(m)}} - 1\right).$$  \hspace{1cm} (1)

**Theorem 1.1.** If there exists some $\bar{\alpha} < \infty$ and $\bar{\beta} < \infty$ such that $\forall m$, $\alpha^{(m)} < \bar{\alpha}$ and $\beta^{(m)} < \bar{\beta}$, then $\lim_{m \to \infty} p(K^{(m)} = 0) = 1$.

**Proof.** Let $\mathbb{N}^+$ be the positive integers. The $\mathbb{N}^+$ are a communicating class for the Markov chain (it is possible to reach any member of the class from any other member) and each $K^{(m)} \in \mathbb{N}^+$ has a nonzero probability of transitioning to the absorbing state $K^{(m+1)} = 0$, i.e., $p(K^{(m+1)} = 0 \mid K^{(m)} > 0) > 0$, $\forall K^{(m)}$. If, conditioned on nonabsorption, the Markov chain has a stationary distribution (is quasi-stationary), then it reaches absorption in finite time with probability one. This is the requirement that, conditioned on having not yet reached a restaurant with no dishes, the number of dishes in deeper restaurants will not explode.

The quasi-stationary condition can be met by showing that $\mathbb{N}^+$ are positive recurrent states. We use the Foster–Lyapunov stability criterion (FLSC) to show positive-recurrency of $\mathbb{N}^+$. The FLSC is met if there exists some function $\mathcal{L}(\cdot) : \mathbb{N}^+ \to \mathbb{R}^+$ such that for some $\epsilon > 0$ and some finite $B \in \mathbb{N}^+$,

$$\sum_{k = 1}^{\infty} p(K^{(m+1)} = k \mid K^{(m)}) \left(\mathcal{L}(k) - \mathcal{L}(K^{(m)})\right) < -\epsilon \text{ for } K^{(m)} > B$$  \hspace{1cm} (2)

$$\sum_{k = 1}^{\infty} p(K^{(m+1)} = k \mid K^{(m)}) \mathcal{L}(k) < \infty \text{ for } K^{(m)} \leq B.$$  \hspace{1cm} (3)

For Lyapunov function $\mathcal{L}(k) = k$, the first condition is equivalent to

$$\left(\alpha^{(m)} \sum_{k = 1}^{K^{(m)}} \frac{\beta^{(m)}}{k + \beta^{(m)}} - 1\right) - K^{(m)} < -\epsilon.$$  \hspace{1cm} (4)

We observe that

$$\alpha^{(m)} \sum_{k = 1}^{K^{(m)}} \frac{\beta^{(m)}}{k + \beta^{(m)}} - 1 < \bar{\alpha} \sum_{k = 1}^{\infty} \frac{\bar{\beta}}{k + \bar{\beta} - 1},$$  \hspace{1cm} (5)
for all $K^{(m)} > 0$. Thus, the first condition is satisfied for any $B$ that satisfies the condition for $\bar{\alpha}$ and $\bar{\beta}$. That such a $B$ exists for any finite $\bar{\alpha}$ and $\bar{\beta}$ can be seen by the equivalent condition

$$\left( \bar{\alpha} \sum_{k=1}^{K^{(m)}} \frac{\bar{\beta}}{k + \bar{\beta} - 1} \right) - K^{(m)} < -\epsilon \text{ for } K^{(m)} > B. \quad (6)$$

As the first term is roughly logarithmic in $K^{(m)}$, there exists some finite $B$ that satisfies Eqn 6. The second FLSC condition is trivially satisfied by the observation that Poisson distributions have a finite mean. \qed